PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fold Shapes as Functions of Progressive Strain

P. R. Cobbold

Phil. Trans. R. Soc. Lond. A 1976 **283**, 129-138
doi: 10.1098/rsta.1976.0073

Phil. Trans. R. Soc. Lond. A. **283**, 129–138 (1976) [129]
Printed in Great Britain

Fold shapes as functions of progressive strain

By P. R. Cobbold†
Department of Earth Sciences, University of Leeds, Leeds LS2 9JT

[Plates 6 and 7]

The folding of the components (layers or texture) of a rock system is viewed as an unstable strain-dependent process. The folds undergo successive stages of development, including initiation, amplification, propagation and decay. Fold shapes are functions of (i) initial morphology, (ii) mechanical behaviour of the rock, including stiffness contrasts and frictional properties of adjacent components, (iii) overall finite strain. The folded components may or may not adopt periodic waveforms, depending on (i) the relative rates of propagation versus amplification of the folds and (ii) the boundary conditions of the rock system.

Introduction

Tectonic folding of rock components is probably an unstable process involving progressive changes in the growth rate of folds and in the stress levels required for deformation. Analogous unstable processes are known to be responsible for the formation of many geological and non-geological deformation structures (e.g. dislocations, fractures, kink-bands) and may be responsible for others (e.g. shear zones, stylolites, zones of pressure-solution).

The object of this paper is to study how the folding process evolves in a layered rock undergoing a progressive overall strain. Data for the study are obtained from (*a*) physical models deformed in laboratory experiments, (*b*) folds formed by natural tectonic processes and (*c*) existing theories of unstable deformations. By studying folding in the light of the known behaviour of other unstable systems, it may be possible to provide a useful link between experimental, theoretical and field data.

Instability in general

In general the rate of a physical process may be directly dependent on the intensity of a 'driving force' (e.g. stress, temperature, voltage). At a certain stage in its development, it may be possible for the process to continue under a less intense driving force. If so there will generally be an increase in the rate of the process. This instability may continue for a finite interval of time and may even be self-accelerating or 'explosive'.

In deforming materials, the driving force can be a certain level of applied stress. For deformation to accelerate, the material must undergo some form of strain-softening, so that it is less resistant to deformation as the deformation proceeds. Strain-softening is therefore a cause of instability. Should the material maintain its strength or strain-harden, the deformation will be stable.

† Present address: Institut de Géologie, Université de Rennes, Avenue Général Leclerc, 35031 Rennes, France.

In most deformations, strain-softening occurs over a limited but finite interval of deformation, i.e. it sets in, persists, and finally disappears (the material hardening again). As a result, the instability itself goes through successive stages of (i) onset, birth or initiation, (ii) growth or amplification, (iii) decay. Finally, the process may stabilize (figure 1*a*).

In a deforming material, the strain-softening and resulting instability need not occur simultaneously over the whole volume considered. It may instead start (nucleate) in one determined site and subsequently spread (propagate) into the surrounding material. If the rate of propagation is faster in certain directions than it is in others, the resulting structure will be heterogeneous, zones of relatively high strain alternating with zones of relatively low strain. This heterogeneity is a characteristic feature of unstable deformations. Also, if nucleation occurs at more than one site, there may be a later interference between propagating zones of instability.

All these time and space effects can occur in the evolution of deformation structures such as fractures (Jaeger & Cook 1971, p. 329) and dislocations (Hull 1969). They can be applied also to folds.

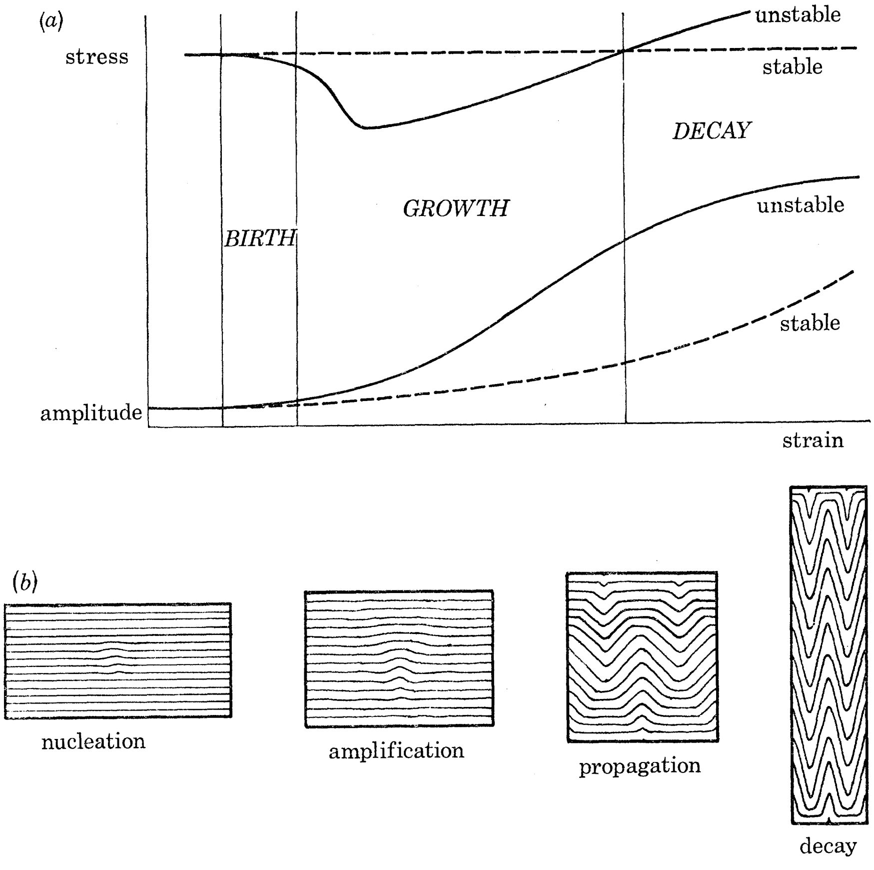


FIGURE 1. (*a*) Stress versus strain and amplitude versus strain for stable and unstable deformations. (*b*) Successive stages of fold development.

FOLDING AS AN INSTABILITY

In many tectonic folds, it is a compositional layering, a texture or a series of parallel discontinuities that has been folded. These components have mechanical properties and therefore tend to be active elements in a deformation as opposed to merely passive markers (Donath &

Parker 1964). To a first order of approximation, all three kinds of layering (figure 2) can be represented by homogeneous material of anisotropic rheology (Biot 1965; Bayly 1970; Cobbold, Cosgrove & Summers 1971). This property they share. To a second order, each kind of layering differs somewhat from the homogeneous model.

Rheological anisotropy is a fundamental mechanical property of a layering and is likely to be an important cause of folding. Each kind of layering therefore folds for the same underlying reason, even though there may be second-order differences in fold shape resulting from the differences between the kinds of layering.

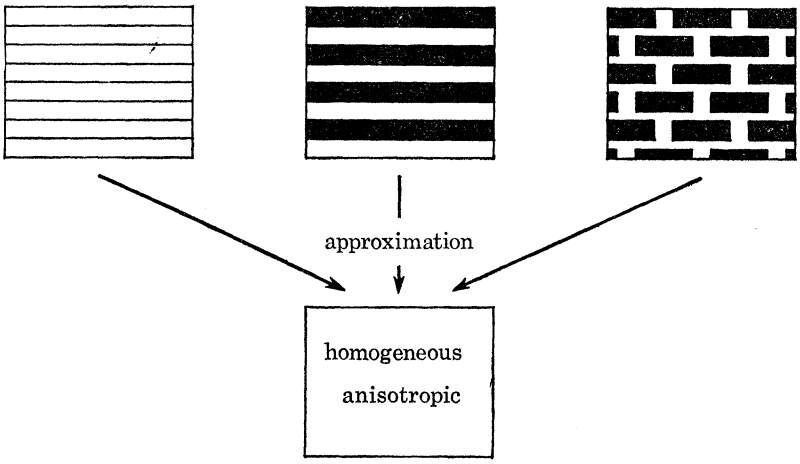


FIGURE 2. Three kinds of layering approximated as a homogeneous anisotropic material.

By its very definition (Ramsay 1967), folding implies some rotation of the layering and this rotation may reach large finite values (90° or more). For a rotating anisotropic material, the resistance to shortening along a fixed direction in space may change greatly (Biot 1965; Bayly 1970). A continuous rotation of the layering in one sense can produce, first, a marked decrease in compressive resistance and, second, an increase. This represents a form of strain-softening, followed by a hardening, which are the conditions for onset, growth and decay of instability. Of course any other mechanisms of strain-softening that may be present (e.g. microfracturing, cataclasis, dislocations and pressure-solution) will also affect the stability of the system.

Rotation of the layering can occur for geometric and for mechanical reasons. Geometrically, any line initially oblique to the principal strain directions rotates towards the principal extension during deformation. The mechanical reason is that stress trajectories tend to refract in an anisotropic material: the principal compressive stress direction becomes nearly parallel or normal to the layering (Donath 1968; Cobbold *et al.* 1971; Treagus 1973). This refracted stress can have a component that assists rotation of the layering.

Because folding is an unstable process, it can go through successive stages of nucleation, amplification, propagation and decay (figure 1*b*). Nucleation (Paterson & Weiss 1966) involves rotation of the layering at selected sites where there are heterogeneities in the deformation. A heterogeneity may be inherent (e.g. an initial variation in composition, thickness or orientation of the layering) or it may be generated (e.g. by a vibration, or by a local fluctuation in applied boundary stresses). The importance of heterogeneities to folding has been emphasized by Willis (1894), Biot (1961), Paterson & Weiss (1966), Johnson (1969) and Cobbold *et al.*

(1971). A similar importance has been attributed to heterogeneities in the nucleation of fractures (Jaeger & Cook 1971) and dislocations (Hull 1969).

Amplification is the progressive growth in the amplitude of a fold at any one point in the folded layering. Propagation involves a spreading of the area of folding, which gradually encroaches into neighbouring areas of previously unfolded layering. The last stage of decay is reached when the layering at any one point has rotated into a nearly stable position and the resistance to deformation has increased. This is often referred to as 'locking up'.

The four stages of nucleation, amplification, propagation and decay will be described in more detail with reference to two kinds of geologically relevant system, a single layer embedded in a matrix and a multilayer.

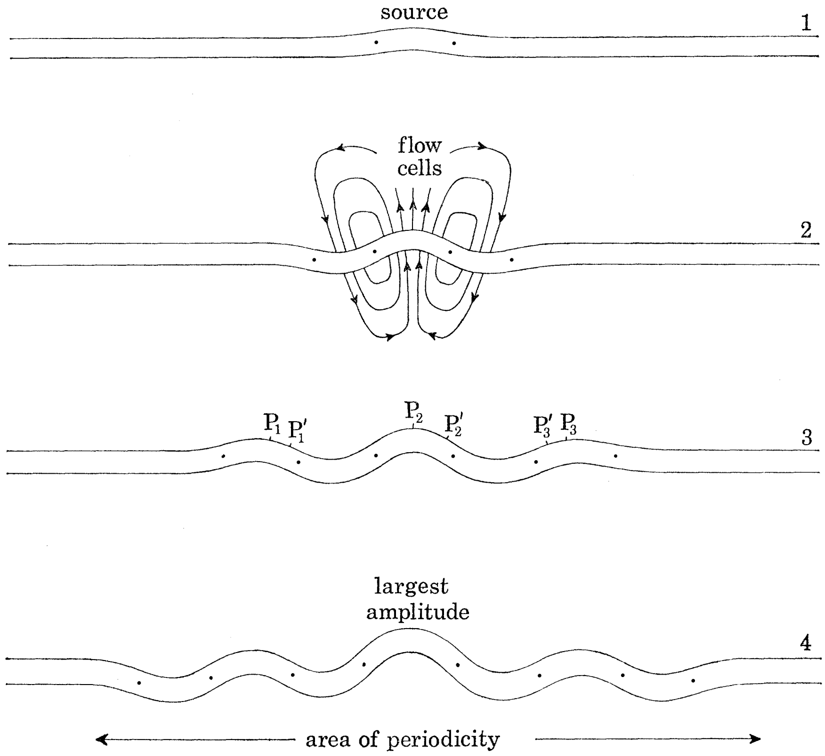


FIGURE 3. Four successive stages in the buckling of a single embedded layer.

BUCKLING OF A SINGLE EMBEDDED LAYER

Some experiments on the buckling of a single layer of stiffer material embedded in a matrix of softer material have been carried out (figure 3 and Cobbold 1975). The materials used (paraffin waxes) were rheologically similar to many rocks. The wax used for the layer was ten times stiffer than that used for the matrix and the compression direction was parallel to the initial orientation of the layer. The layer was initially of constant thickness and planar except at one central site where there was a deflection of small amplitude.

As the model deformed, buckling initiated at the central heterogeneity, producing an antiform (figure 3). The antiform amplified and simultaneously two flanking synforms appeared. These three folds then continued to amplify, causing a cell-like flow in the matrix. The rate of amplification depended on the shape of the initial deflection. In some experiments no further new folds appeared, but in others (depending on the shape of the initial deflexion) new folds appeared serially (Price 1967) in time and distance along the layer. The buckling therefore propagated along the layering. It is believed that such propagation is a result of flow in the matrix, the propagation rate depending on the dimensions of the flow-cells. If the propagation rate is slow compared with the amplification rate of the folds, the resulting structure is a complex (Erzhanov & Egorov 1970) containing a minimum of three adjacent folds. If the propagation rate is relatively fast, the resulting complex may contain many individual folds and therefore acquires a regular spatial periodicity and a sinusoidal shape. Formation of such a regular complex may require an overall strain of large finite value (15 % or more).

Complexes containing three or more individual fold members can be identified in naturally deformed embedded layers (Cobbold 1975).

Buckling of a multilayer

For a single embedded layer, instability involves the layer itself and a zone of contact strain in the matrix (Ramberg 1962) where the flow-cells develop. For a multilayer, in contrast, instability may extend to most, if not all, of the layers (figure 4, plate 6).

The instability can nucleate at any heterogeneity, such as an initial deflexion in the layering (figure 4) or a boundary disturbance. The subsequent stages of fold growth, including amplification and propagation, will be described in more detail.

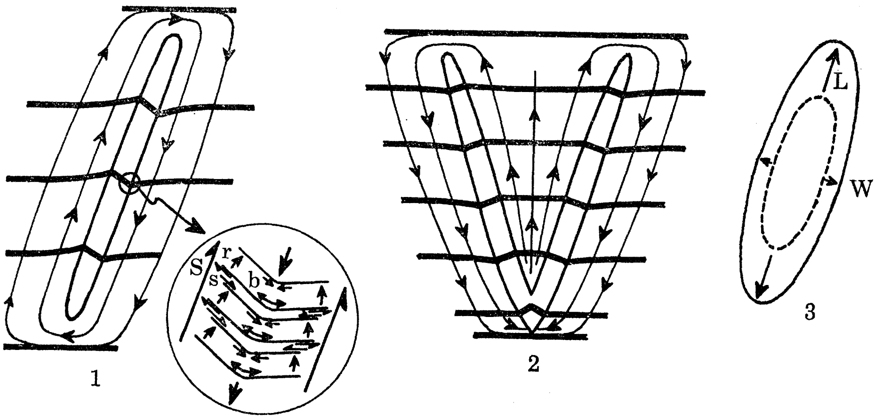


Figure 5. Flow-cells in a buckling multilayer. 1. Single flow-cell. Flow-lines define a shear (S, inset), with microcomponents of rotation (r), flexural flow or slip (s) and bending (b). 2. Symmetrically paired (conjugate) flow-cells. 3. Propagation of a flow-cell by lengthening (L) and by widening (W).

Amplification

No single folded layer in the sequence can amplify without deflecting the layers immediately above and below it. For this reason amplification must occur simultaneously, folds of a specific size and shape including a certain minimum number of layers. Also, unless there are very large volume changes in the material, the minimum number of folds that may amplify simultaneously is two (figure 5). The flow-lines in the material define a cell. To a first approximation, the flow pattern over most of one cell is a shear parallel to the axial plane of the folds. On a microscale, this may resolve into components of bending, rotation and flexural flow or slip (inset, figure 5).

Experiments with physical models suggest that flow-cells in multilayers are generally ellipsoidal rather than spherical. In plane strain deformations (producing cylindrical folds), the long axis of a flow-cell is generally oblique to the layering and to the principal compression direction. The long axis apparently represents a direction of 'easy shear' in the multilayer (Biot 1965; Cobbold *et al.* 1971; Johnson & Ellen 1974). In some situations, flow-cells form with a unique long-axis direction (figure 5), whereas in other situations, the flow-cells may be paired and there are two conjugate long-axis directions. Paired flow-cells are responsible for the amplification of three adjoining folds.

During amplification, experimentally formed folds may attain large amplitudes without forming part of a periodic complex. This feature can also be seen in some folds formed by natural processes (figure 6, plate 6).

Propagation

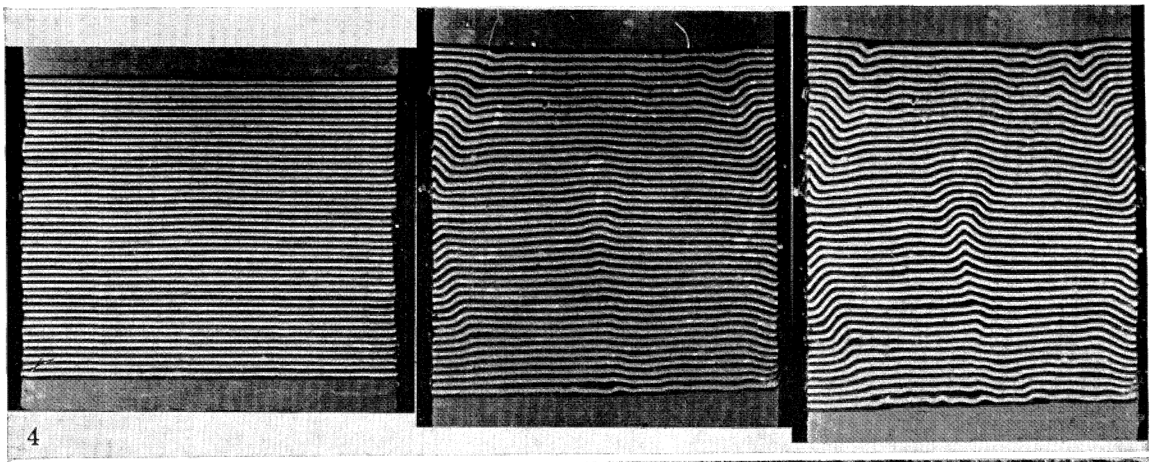
In many experiments with physical models, flow-cells do not remain static but change in size and shape. In general the flow-cells increase their dimensions as the deformation proceeds. The zone of folding therefore increases in size, encroaching on surrounding areas which previously were unfolded. The flow-cells can be said to propagate.

Propagation can occur in all directions simultaneously, but generally it is fastest in one characteristic direction approximately parallel to the long axis of the flow-cell. With increasing deformation, the flow-cells therefore become more elongate (figure 5) and an increasing number of layers are involved in the folding.

If propagation occurs in the short-axis direction of the flow-cell, the fold hinges migrate along the layering (Ramsay 1967, p. 452). This process requires (i) a straightening of those segments of layering that were previously bent and (ii) a bending of those that were previously straight. The straightening implies a local reversal in the deformation history. Experimentally such reversals are easy to detect and are relatively common (figure 4, Paterson & Weiss 1966; Weiss 1968; Cobbold *et al.* 1971; Gay & Weiss 1974). There is some argument as to whether they occur in natural processes (Gay & Weiss 1974). One way of detecting reversals is to look for associated microstructures (e.g. fractures, cleavages, slickensides) that form irreversibly. Some fold hinges, for example, develop triangular zones of dilation at the outer arcs (figure 7, plate 7). Commonly these zones are infilled with crystalline calcite or quartz. In some examples, the present fold axial surfaces coincide with arrays of dilation zones; in other examples there is no coincidence, suggesting that hinge-migration has occurred in the rock.

Reflexion

As a multilayer fold propagates it may encounter a change in the average properties of the multilayer, i.e. an internal or external boundary. The boundary may offer a resistance to transverse flow, either because it is relatively rigid, or because it facilitates tangential displacements. In this situation the fold ceases to propagate as before. Where there are conjugate directions of maximum propagation rate, the fold may reflect from one direction to the other (figure 8, plate 7). With increasing deformation it may be possible for the fold to reflect several times in succession at two or more boundaries. If the boundaries are parallel and planar, the successive reflexions result in a complex of adjoining folds (figure 9*a*). The complex has a regularly periodic shape with a wavelength determined by (i) the angle of incidence, (ii) the angel of reflexion and (iii) the distance between the reflecting surfaces. Geologically, reflecting surfaces of this kind may occur at the internal boundaries between lithological units (Currie, Patnode & Trump 1962) of different average composition. Regularly periodic multilayer folds often may be found confined to such units (Currie *et al.* 1962; Cobbold *et al.* 1971; Johnson & Ellen 1974).



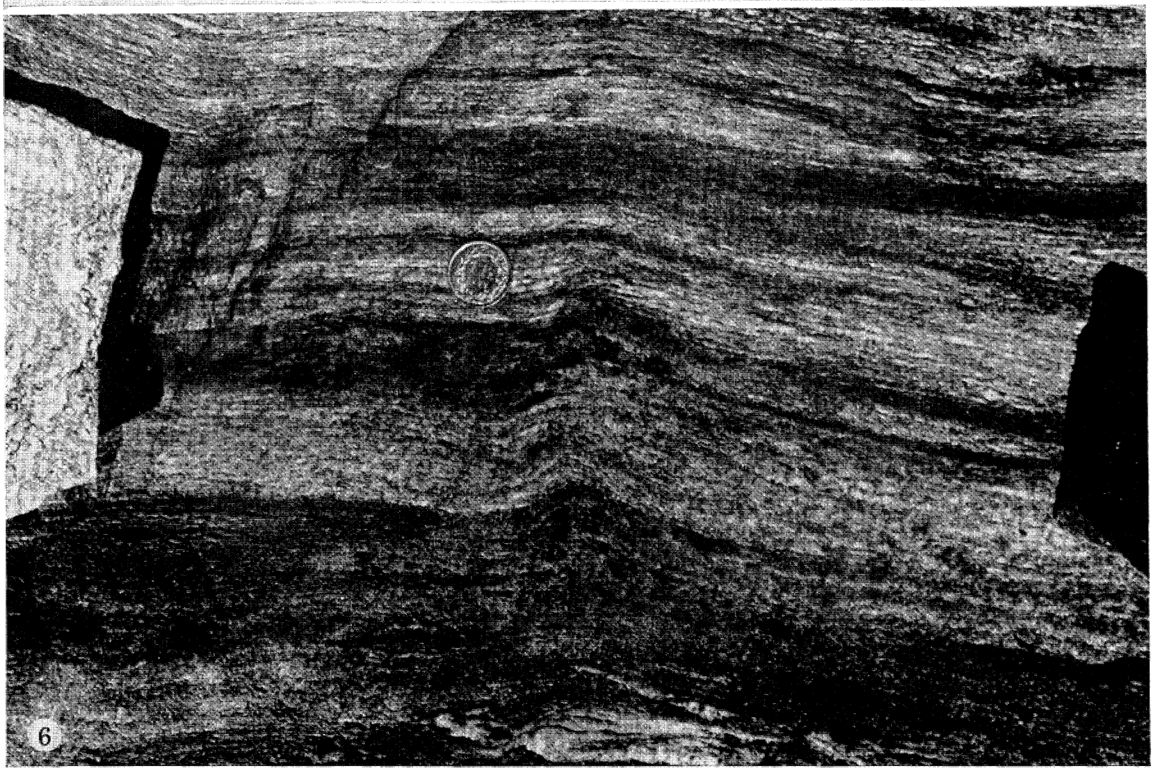
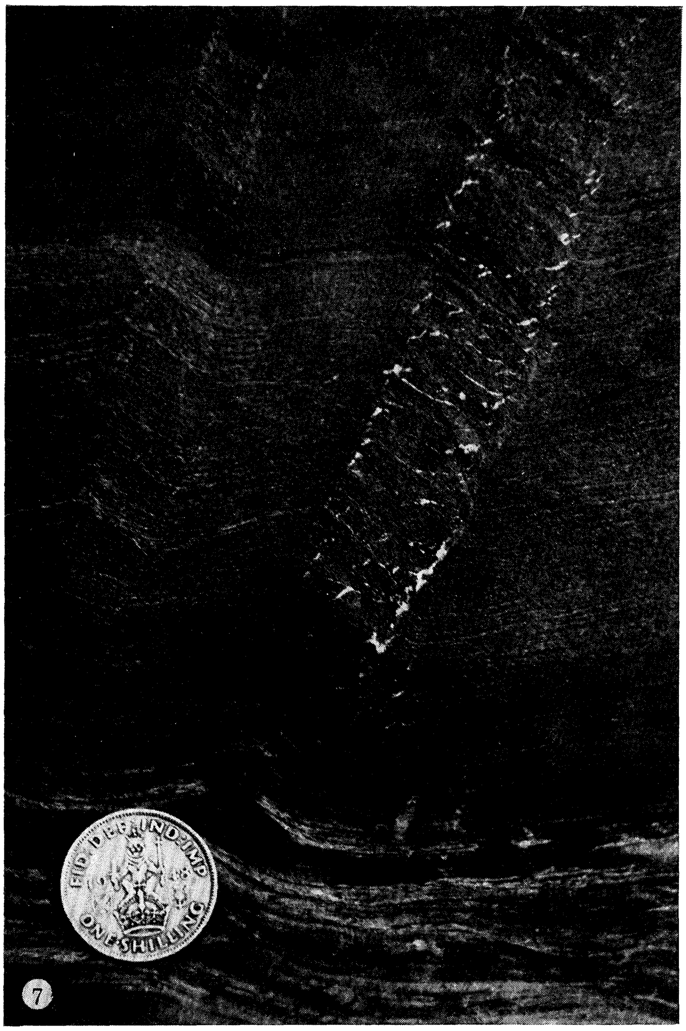
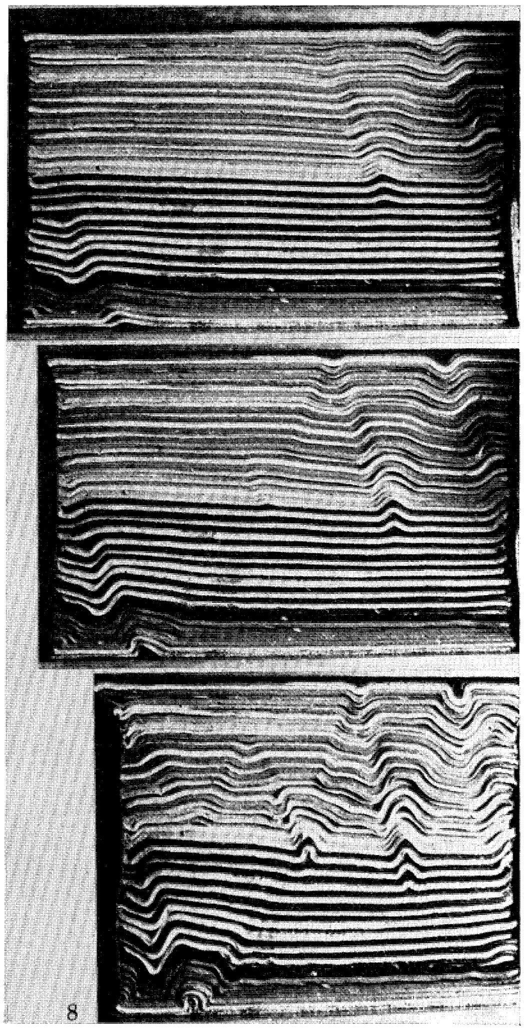


FIGURE 4. Three successive stages in the internal buckling of multilayer model M1. Folds nucleated in the central area of the model (initial deflexion in the layering) and at the boundaries (stress heterogeneity due to friction).

FIGURE 6. Isolated fold complex in a multilayered granitic gneiss, Cristallina, Ticino, Switzerland. The complex may have nucleated at an initial heterogeneity (intrafolial isocline, I, from an earlier phase of folding).

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES





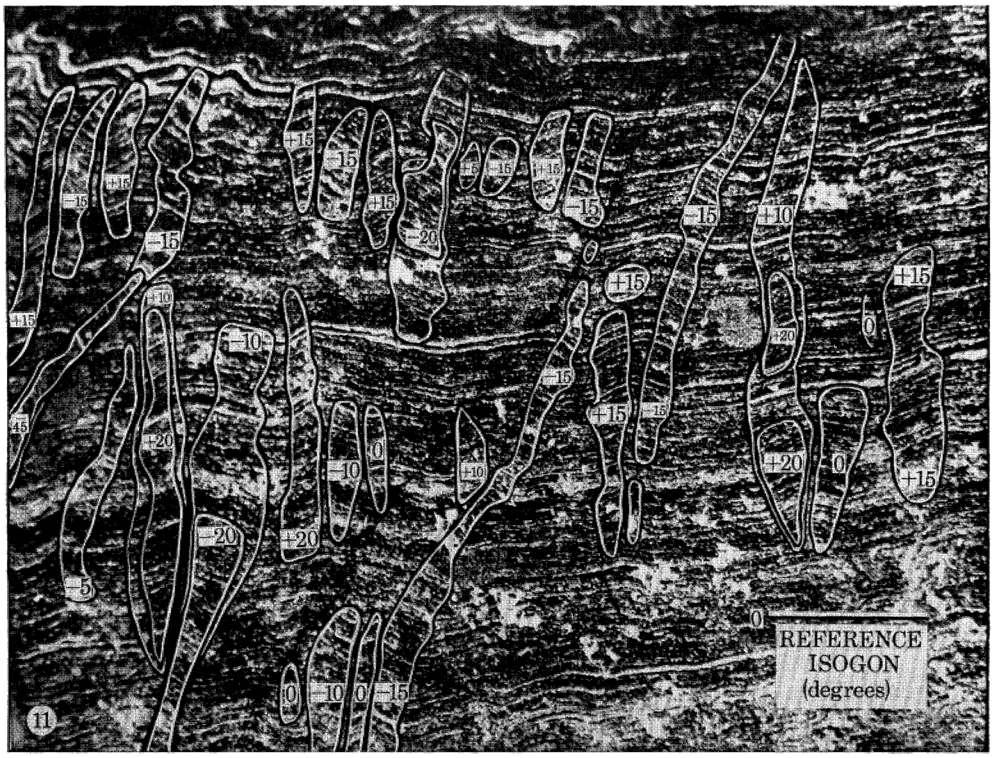


FIGURE 7. Evidence for hinge migration in phyllites, south Devon, England.

FIGURE 8. Three successive stages in the internal buckling of multilayer model M2.

FIGURE 11. Folds in a layered amphibolite, Loch Hourn, Scotland (photograph by A. J. Watkinson). The contours are fold isogons.

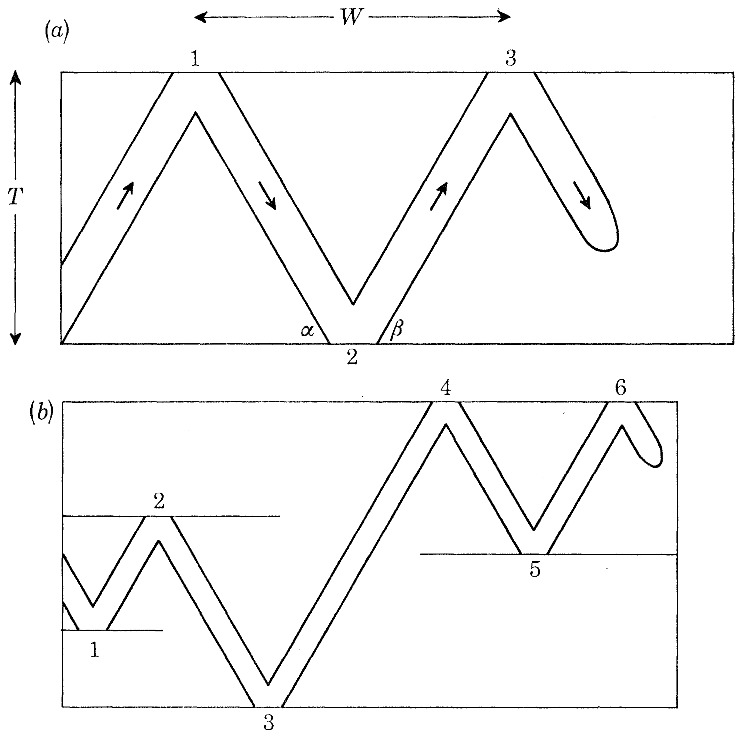


FIGURE 9. (*a*) Formation of a periodic complex by successive reflexion between parallel boundaries. (*b*) Staggered reflexion.

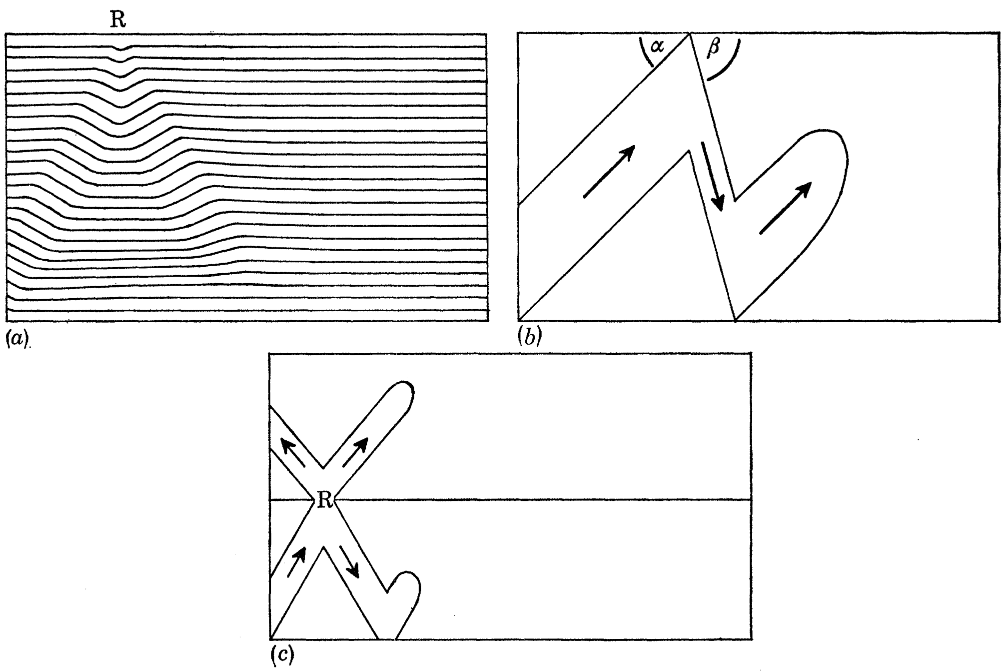


FIGURE 10. Reflexion and refraction of folds. (*a*) Reflexion at a point, *R*, on the rigid external boundary of a multilayer. (*b*) Reflexion where the applied stress is obliquely inclined to the layering. The angle of incidence, α, is smaller than the angle of reflexion, β. (*c*) Simultaneous reflexion and refraction at a point, *R*, on an internal boundary.

Internal boundaries between multilayer units may act as sources not only of reflexion but also of refraction (figure 10). This is because the direction of maximum propagation depends on the rheological properties of the multilayer unit.

If some internal boundaries do not persist laterally, reflexion may be 'staggered' and the resulting fold complex does not have a regular wavelength (figure 9*b*).

Interference

In a multilayer of relatively large total thickness and lateral extent, nucleation may commence at various sources and at different times. Propagation from each source may form local fold complexes, each with its own regular periodicity if there are suitable reflecting horizons. As the complexes spread, they will tend to interact, either linking up or interfering, depending on their relative spacing and wavelengths. If two complexes are spaced an exact number of wavelengths apart, they will be in phase upon meeting and therefore will link up (constructive interference). If they are not an exact number of wavelengths apart, the complexes may interfere destructively, i.e. each stops the other from propagating. The resulting fold structure will contain two areas of regular periodicity, separated by an area of no folding or of irregular periodicity.

In some situations, two complexes may interfere constructively, even though the spacing is not an exact number of wavelengths. This they do by meeting at a point within the multilayer unit, while maintaining the characteristic angle of propagation (figure 8). The resulting wavelength is irregular.

In other situations propagating folds may cross-cut one another (Paterson & Weiss 1966). Points of intersection may act as sources for the subsequent nucleation of further folds.

The distribution and morphology of natural examples of folded multilayered rocks are often irregular at first sight (figure 11). A convenient way of analysing these folds is to draw dip isogons (Elliot 1965; Summers 1970). These emphasize the relative rotations undergone by adjacent segments of the layering. In experimental folds there is a close correspondence between the shapes of flow-cells and of fold isogons. In natural folds (figure 11) the shape and distribution of isogons and axial planes suggest that folds may have formed by processes of nucleation, amplification, propagation and reflexion.

Discussion

Evidence from folds formed experimentally and naturally suggests that the folding process involves propagation. In direct contrast with the experimental and natural evidence, most existing mathematical theories (Biot 1965; Ramberg 1963, 1970; Johnson & Ellen 1974; Johnson & Honea 1974) are based on the assumption that folds appear (*a*) instantaneously, (*b*) within one small increment of deformation, (*c*) throughout all the rock system at the same time, and (*d*) with a 'naturally inbuilt' regular periodicity. These assumptions initially arose from considering the rock as an elastic material. The assumptions may not be so correct in situations where (*a*) there are initial heterogeneities or (*b*) there is viscous deformation or creep. Here the propagation rate may be slow compared with the rate of amplification, so that resulting folds attain large amplitudes without necessarily forming part of a periodic complex.

Theory and experiment may be reconcilable in some respects. For example, theories of deformation and folding of anisotropic materials (Biot 1965; Cobbold *et al.* 1971; Johnson &

Ellen 1974) predict the formation of adjacent areas of high and low intensity of deformation, bounded by theoretical discontinuities known mathematically as characteristic lines. The characteristic lines can be correlated with the directions of maximum propagation rate in experimentally deformed models. The theory predicts what would happen if propagation were infinitely rapid, leading to an equilibrium configuration. In the models, propagation occurs at a finite speed.

Similar effects have been observed in other kinds of deformation (and indeed in other physical processes). For example, mathematical theories of plastic flow in metals (Nadai 1950) predict that zones of high deformation (Lüders bands, deformation bands or shear zones) will be separated from zones of no plastic deformation (dead metal zones) by characteristic lines that are parallel to the direction of maximum shear stress. In laboratory experiments on metals, it may be observed that zones of shear (i) nucleate at heterogeneities and (ii) propagate fastest along the shear direction (Hull 1969).

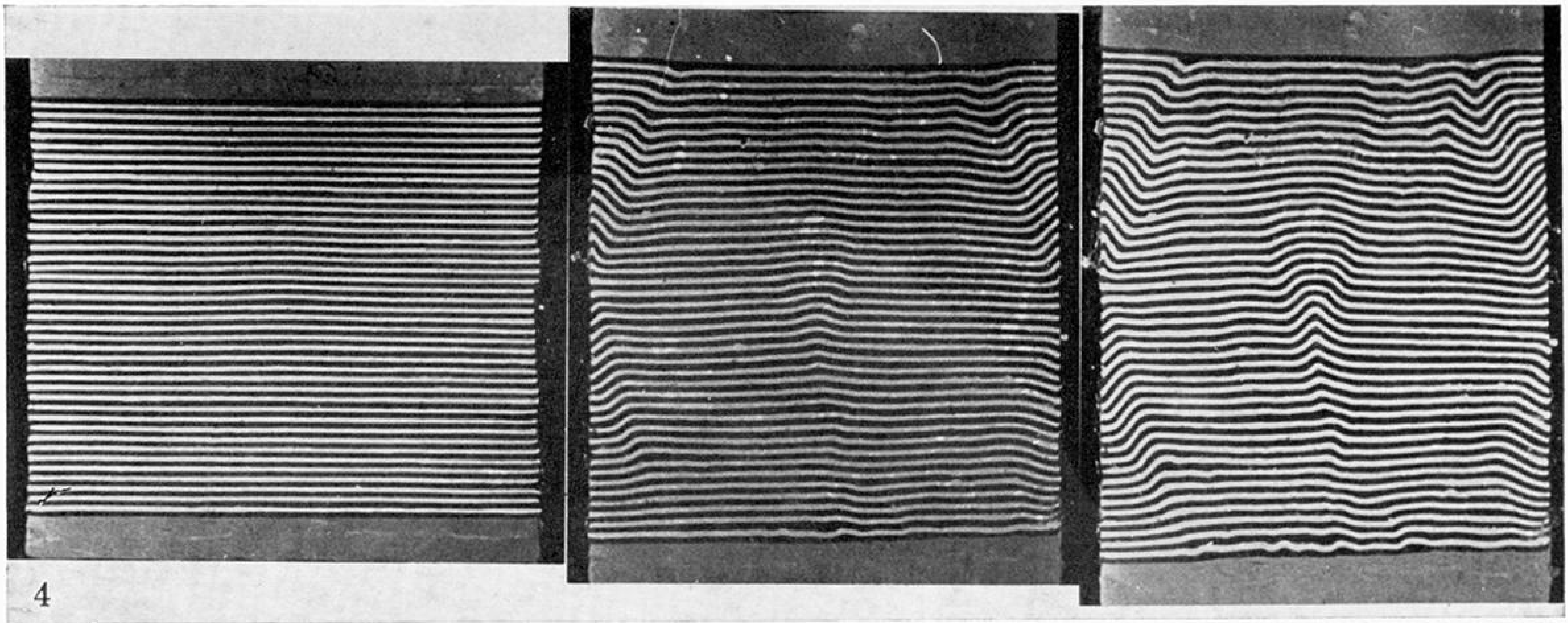
The existing theories of folding may provide approximate predictions of the wavelength or axial surface orientation of folds, where the propagation rate has been rapid. What the theories cannot predict is the spatial distribution of folds or the regularity of their spatial periodicity. For the latter features one should perhaps look to experimental work, observation of natural folds, or more advanced mathematical methods that can deal with the progressive incremental history of unstable deformation.

References (Cobbold)

Bayly, M. B. 1970 Viscosity and anisotropy estimates. *Tectonophysics* **9**, 459–474.
Biot, M. A. 1961 Theory of folding of stratified viscoelastic media. *Geol. Soc. Am. Bull.* **72**, 1595–1620.
Biot, M. A. 1965 *Mechanics of incremental deformations*. New York: Wiley.
Cobbold, P. R. 1975 Fold propagation in single embedded layers. *Tectonophysics* **27**, 333–351.
Cobbold, P. R., Cosgrove, J. W. & Summers, J. M. 1971 Development of internal structures. *Tectonophysics* **12**, 23–53.
Currie, J. B., Patnode, H. W. & Trump, R. P. 1962 Development of folds in sedimentary strata. *Geol. Soc. Am. Bull.* **73**, 655–674.
Donath, F. A. 1968 Experimental study of kink-band development. In *Proc. Conf. on Research in Tectonics* (ed. A. J. Baer & D. K. Norris). *Geol. Surv. Can. Pap.* **68,52**, 255–292.
Donath, F. A. & Parker, R. B. 1964 Folds and folding. *Geol. Soc. Am. Bull.* **75**, 45–62.
Elliot, D. 1965 The quantitative mapping of directional minor structures. *J. Geol.* **73**, 865–880.
Erzhanov, Z. S. & Egorov, A. K. 1970 Mathematical theory of formation of folds. *Proc. 2nd. Cong. Int. Soc. Rock Mechs.* 2–24.
Gay, N. C. & Weiss, L. E. 1974 Relation between principal stress directions and the geometry of kinks. *Tectonophysics* **21**, 287–300.
Hull, D. 1969 *Introduction to dislocations*. London: Pergamon.
Jaeger, J. C. & Cook, N. G. W. 1971 *Fundamentals of rock mechanics*. London: Chapman & Hall.
Johnson, A. M. 1969 Development of folds within Carmel Formation. *Tectonophysics* **8**, 31–77.
Johnson, A. M. & Ellen, S. D. 1974 Theory of concentric, kink and sinusoidal folding. I. Introduction. *Tectonophysics* **21**, 301–339.
Johnson, A. M. & Honea, E. 1974 Theory of concentric, kink and sinusoidal folding. II. Initial stress. *Tectonophysics* **25**, 261–280.
Nadai, A. 1950 *Theory of flow and fracture of solids*. New York: McGraw-Hill.
Paterson, M. S. & Weiss, L. E. 1966 Experimental deformation and folding in phyllite. *Geol. Soc. Am. Bull.* **77**, 343–374.
Price, N. J. 1967 Initiation and development of asymmetric bucklefolds. *Tectonophysics* **4**, 173–201.
Ramberg, H. 1962 Contact strain and folding instability. *Geol. Runds.* **51**, 405–439.
Ramberg, H. 1963 Fluid dynamics of viscous buckling. *Bull. Am. Ass. Petrol. Geol.* **47**, 484–505.
Ramberg, H. 1970 Folding of laterally-compressed multilayers in the field of gravity. I. *Phys. Earth Planet. Int.* **4**, 83–120.

Ramsay, J. G. 1967 *Folding and fracturing of rocks*. New York: McGraw-Hill.
Summers, J. M. 1970 Experiments on the folding of confined multilayers. M.Sc. Thesis, University of London (unpublished).
Treagus, S. H. 1973 Buckling stability of a viscous single-layer. *Tectonophysics* **19**, 271–289.
Weiss, L. E. 1968 Flexural slip folding of foliated model materials. In *Proc. Conf. on Research in Tectonics* (ed. A. J. Baer & D. K. Norris). *Geol. Surv. Can. Pap.* **68,52**, 294–333.
Willis, B. 1894 Mechanics of Appalachian structure. *U.S. Geol. Surv. 13th Ann. Rep.* 1891–1892, 213–281.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES



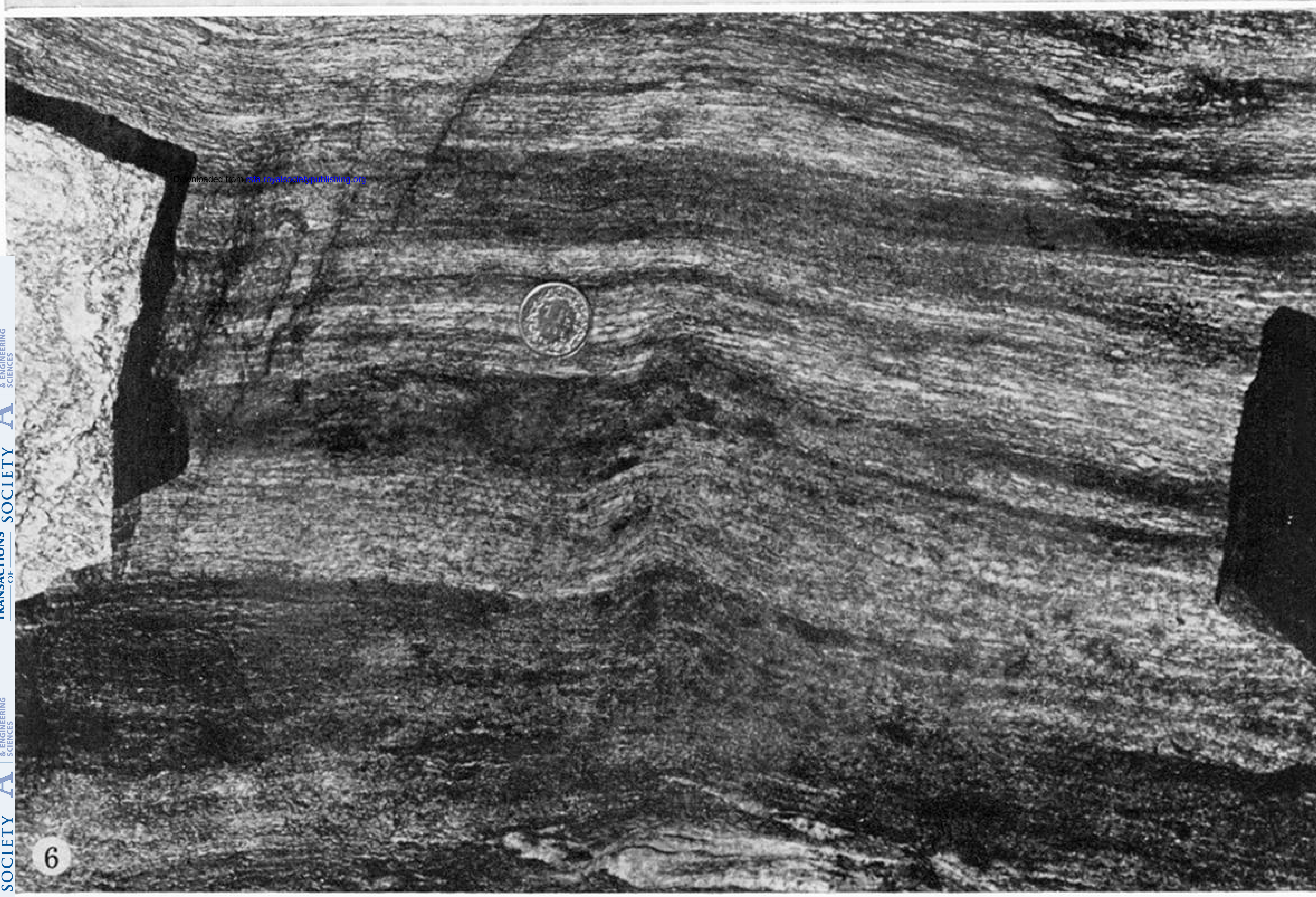
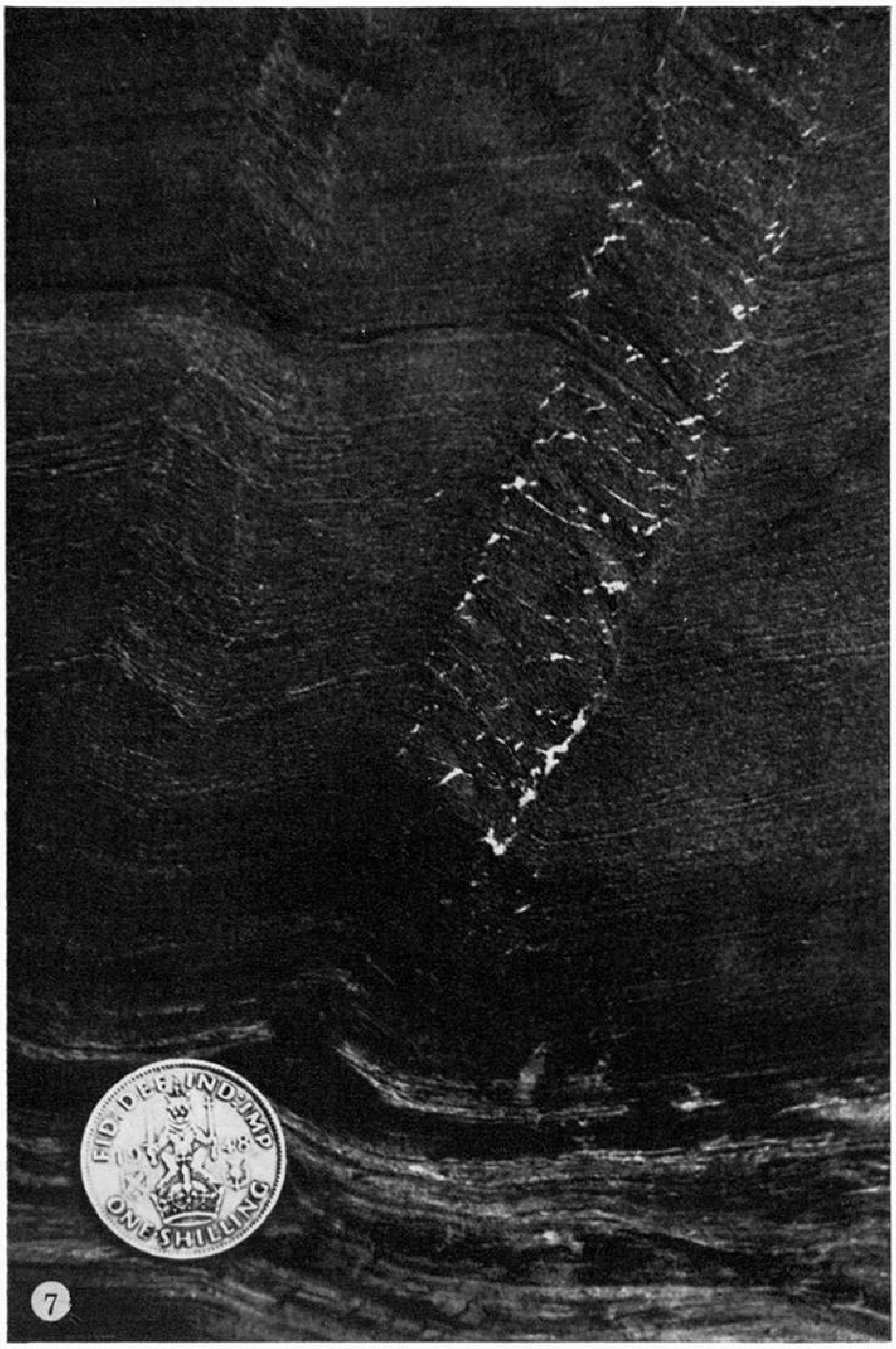
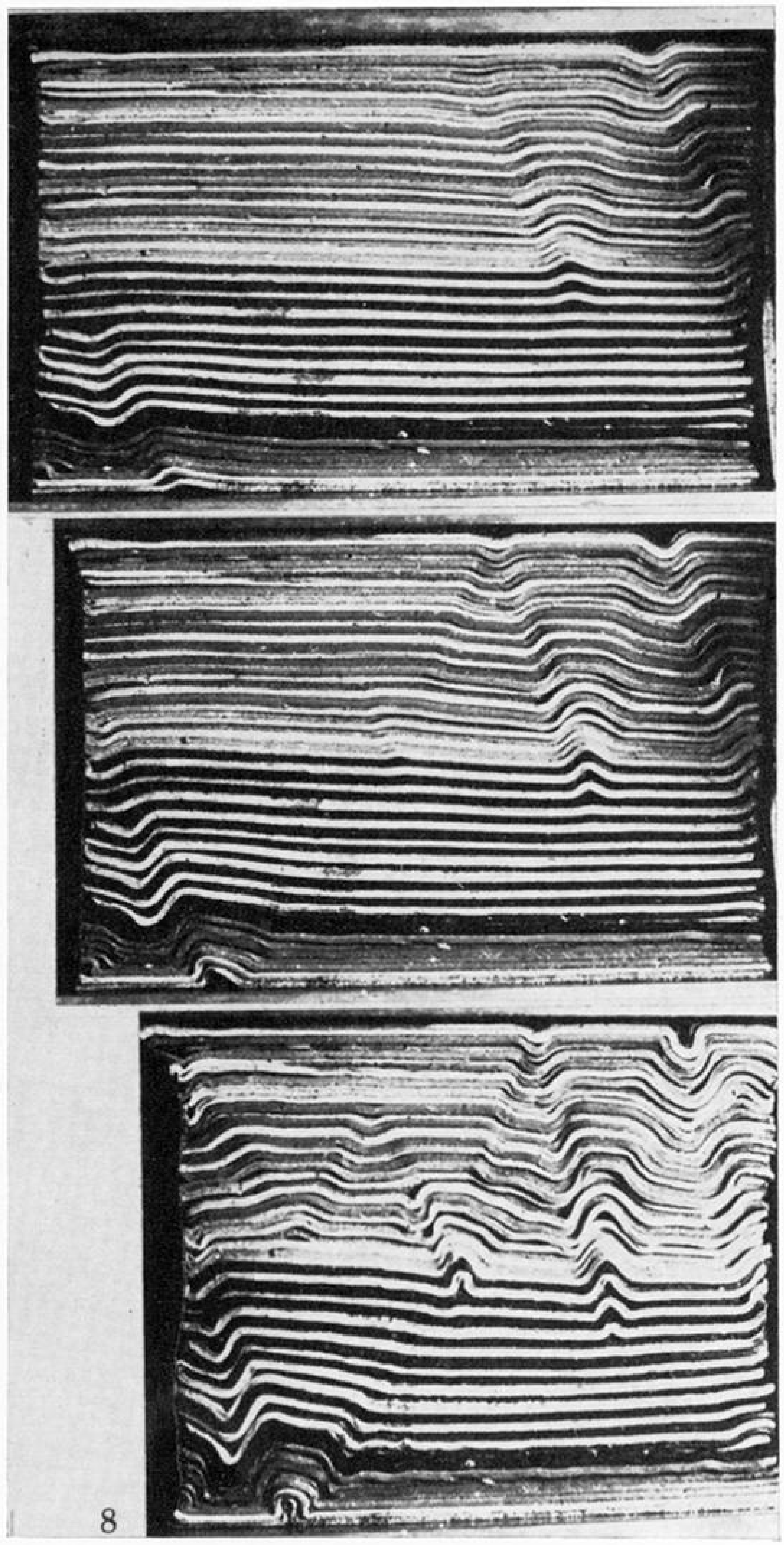


Figure 4. Three successive stages in the internal buckling of multilayer model M1. Folds nucleated in the central area of the model (initial deflexion in the layering) and at the boundaries (stress heterogeneity due to friction).

Figure 6. Isolated fold complex in a multilayered granitic gneiss, Cristallina, Ticino, Switzerland. The complex may have nucleated at an initial heterogeneity (intrafolial isocline, I, from an earlier phase of folding).





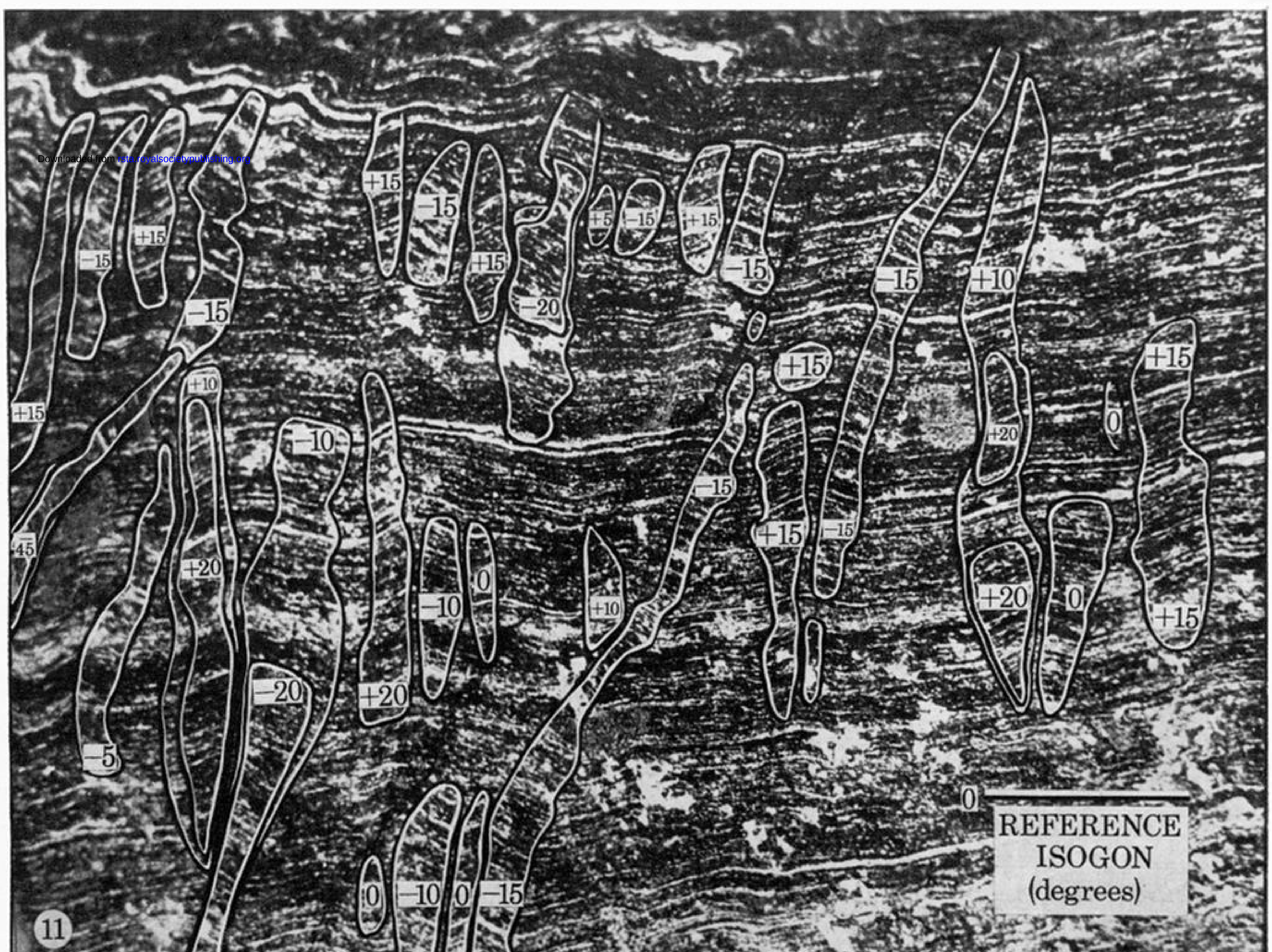


FIGURE 7. Evidence for hinge migration in phyllites, south Devon, England.

FIGURE 8. Three successive stages in the internal buckling of multilayer model M2.

FIGURE 11. Folds in a layered amphibolite, Loch Hourn, Scotland (photograph by A. J. Watkinson). The contours are fold isogons.